Useful Algorithms with Graph Theoretical Concepts in Real Life

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Abstract

In this research paper, the basic definitions of distance-2 dominating sets and domination number are firstly expressed. After that, the algorithms for finding a minimal and minimum distance-2 dominating sets in radio station, computer network and social network are studied. Finally, the applications of distance-2 dominating sets in networks are mainly presented.

Keywords : Dominating set, minimal distance -2 dominating set, minimum distance -2 dominating set.

1. Introduction

Graph theory is an important branch of applied mathematics. It has been increasingly applied to communication networks in real life problems. In this research paper, the useful algorithms of minimal and minimum distance-2 dominating sets in some communication networks were described.

It is considered that the graphs are finite, simple, undirected and weighted graph.

2. Basic Definitions

A subset D of the set of vertices V of a graph G is a **dominating set** of G if every vertex in $V(G)-D$ is adjacent to at least one vertex in D.

The number of vertices of a minimum dominating set of a graph G is called the **domination number** of G and is denoted by $\gamma(G)$.

A set of vertices D in a graph $G=(V(G),E(G))$ is a **distance-2 dominating set** if every vertex $V(G)-D$ is within distance-2 of at least one vertex in D.

The **distance-2 domination number** $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a distance-2 dominating set of G.

A dominating set D is called a **minimal dominating set** if no proper subset of D is a dominating set.

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2.1 Example

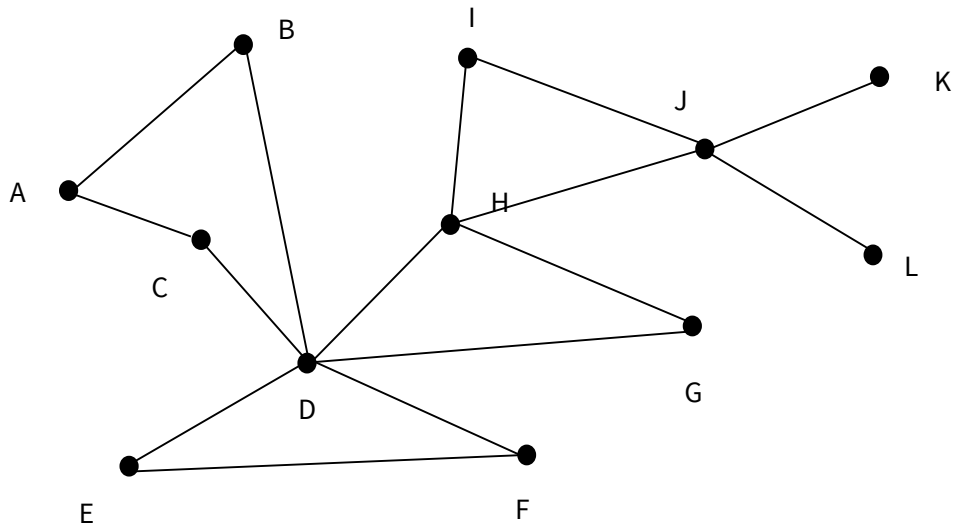

Figure 1. Minimal and minimum distance -2 dominating graph G

In Figure 1, the set $D_1 = \{D, G, I\}$, $D_2 = \{A,B,H,J\}$ and $D_3 = \{D, H\}$ are distance -2 dominating sets. It can be found that the sets D_1 and D_2 are minimal distance -2 dominating sets and D_3 is a minimum distance -2 dominating set and minimum distance -2 domination number, $\gamma_{\leq 2}(G)$ is 2. So, it was noticed that the vertices of minimum distance -2 dominating set have the maximum degree.

3. Algorithms for Minimal and Minimum Distance - 2 Dominating Set

3.1 An Algorithm for Minimal Distance-2 Dominating Set

This algorithm gives a minimal distance -2 dominating set of a graph. The following steps must be performed:

Step 1. All the vertices in V(G) are initialized to white color.

Step 2. Any one vertex in V(G) is chosen and then its colour is changed to red and sends a notification to all its neighbours within the distance two. On receiving this notification, the white colour neighbour vertices within the distance two are turned into green colour.

Step 3. Now, any one white colour vertex is chosen in V (G) which is adjacent to any green colour vertex.

Step 4. The above process (step 2 and 3) is continued until there is no more white colour vertex in the graph.

Step 5. Finally, all the red colour vertices in the graph form a minimal distance -2 dominating set are obtained.

3.2 Example

A minimal distance -2 dominating set can be found in the following graph with twelve vertices by using the above algorithm.

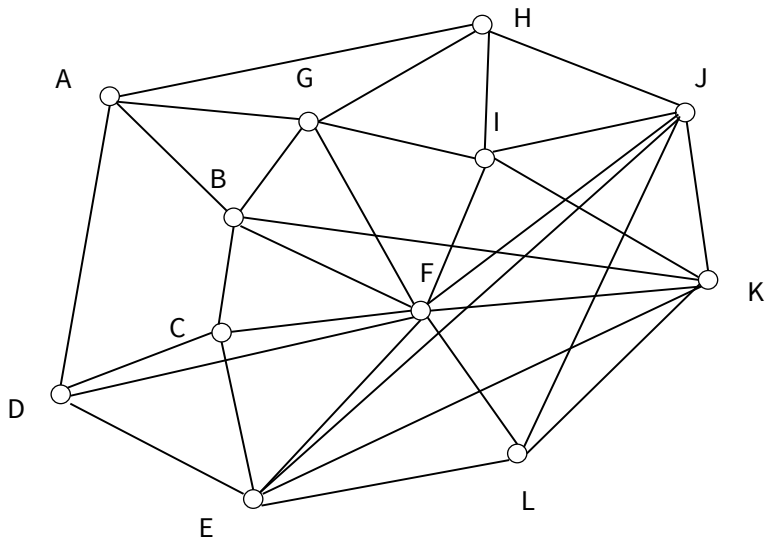

Figure2. Initialization of vertices in the undirected graph G

Firstly, all vertices of graph G are initialized to white colour as shown in Figure 2. Then a vertex A is chosen in V (G) which it is changed to red and its white colour neighbour vertices within the distance two is changed to green colour. And then the vertices B,C,D,E,F,G,H and I are green colour as shown in Figure 3. Next, a white colour vertex J is selected in V(G) which is adjacent to green colour vertex F. After the vertex J is changed to red, its white colour neighbor vertices which is within the distance two are changed to green colour vertices K and L as shown in Figure 4. Now, there is no more white colour vertex in the graph G. Finally, all the red colour vertices in the graph G form a minimal distance -2 dominating set. It was found that the minimal distance -2 dominating set is {A,J} and $\gamma_{\leq 2}(G)$ is 2 as shown in Figure 4.

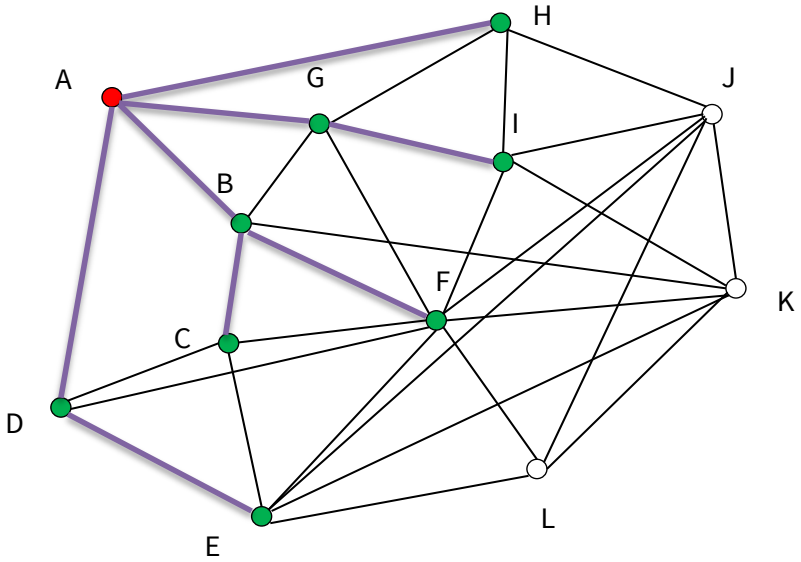

Figure 3. Minimal distance – 2 dominating set

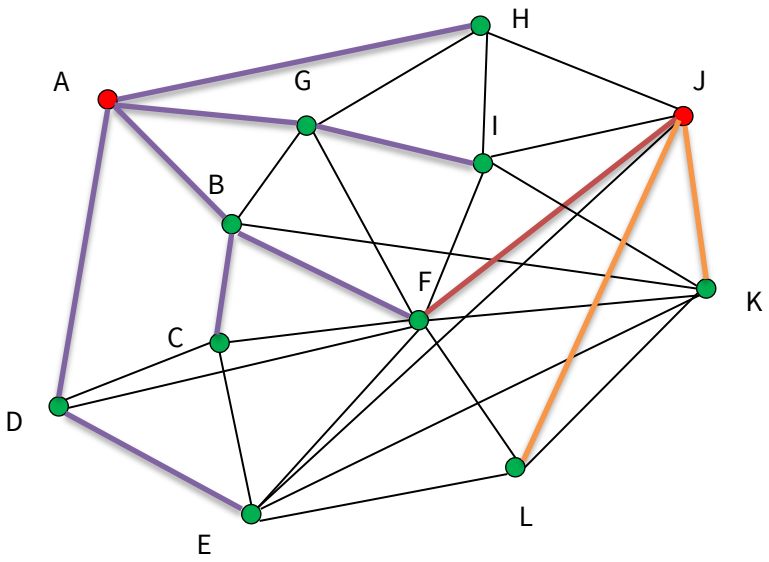

Figure 4. Minimal distance – 2 dominating set

3.3 An Algorithm for Minimum Distance −2 Dominating Set

This algorithm gives the minimum distance -2 dominating set of a graph. The following steps must be performed:

Step 1. All vertices in V(G) are initialized to white colour.

Step 2. A vertex in V(G) which has maximum degree is selected, (if there are two vertices, any one vertex can arbitrarily be chosen). And then its colour is changed to red and a notification is sent to all its neighbours within the distance two. On receiving this notification, the white colour neighbor vertices within a distance two turn into green colour.

Step 3. Now, any one white colour vertex V(G) is chosen.

Case 1. If the white colour vertex has maximum degree, (if there are vertices which are equal maximum degree, any vertices can be chosen) and not adjacent to any green colour vertex in the remaining vertices of V(G).

Case 2. If the green colour vertex which is exactly at the distance two and has more than one pendent vertex, then the green colour vertex is changed into red colour vertex.

Step 4. The above process (step 2 and 3) is continued until there is no more white colour vertex in the graph.

Step 5. Finally, all the red colour vertices are obtained in the graph form a minimum distance -2 dominating set.

3.4 Example

Like the previous graph in Figure 2, it can be considered as follows. A minimum distance -2 dominating set can be found by using above algorithm. Firstly, all vertices are described to white colour as shown in Figure 2. Then, a vertex F is selected in V(G) which has maximum degree and its colour is changed to red and its neighbour vertices within a distance two are turned into green colour vertices A,B,C,D,E,G,H,I,J,K and L as shown in Figure 5. Now, these are no more white colour

vertex in the graph. Finally, it was obtained that the minimum distance -2 dominating set is {F} and $\gamma_{\leq 2}(G)$ is 1 from the graph in Figure 5.

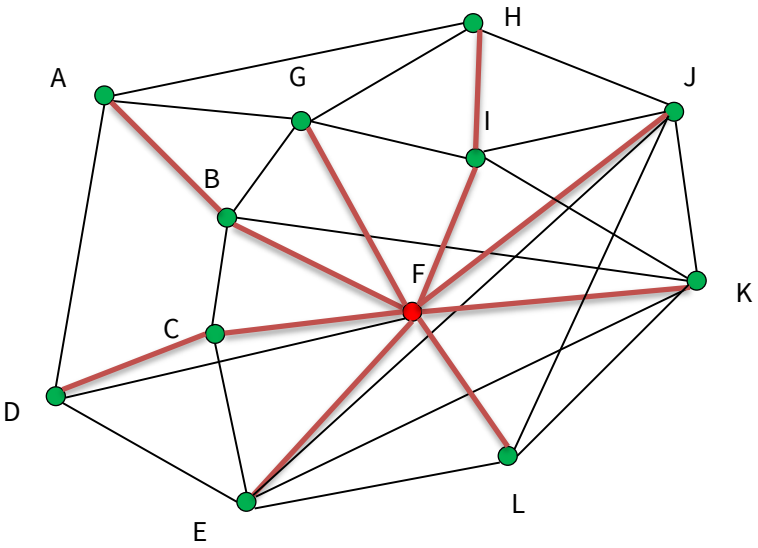

Figure 5. Minimum distance -2 dominating set

4. Applications to Real Life

It was studied to extend the concept of dominating sets to distance-2 dominating sets. It can be applied to many real life problems such as radio stations, computer networks and social networks.

4.1 Radio Station

It was found that the communication network of Mandalay FM radio station. It has a broadcast range of ninety kilometers. It is assumed that there are a collection of towns in the Mandalay region and Sagaing region. Let each town be represented by a vertex. An edge between two towns is linked with the distance. The actual distances between two towns are taken from Google Map. The distance between two towns is shown in Figure 6.

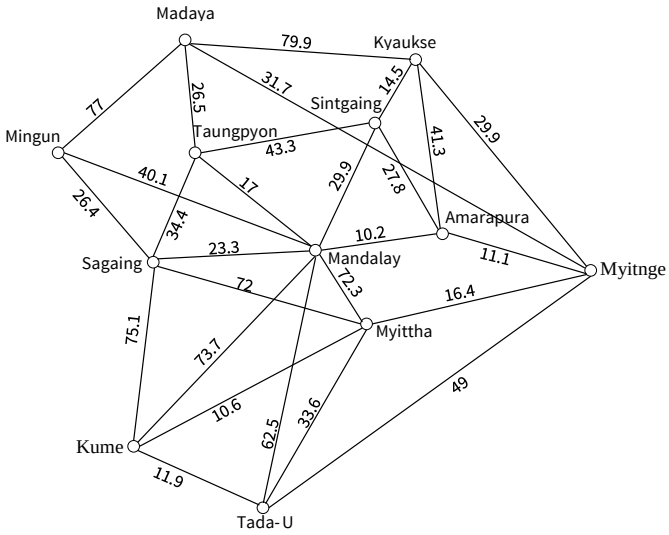

Figure 6. Radio Station of Mandalay FM

Since radio station has a limited broadcasting range, several distances should be used to reach all towns. In this case, a minimal and minimum distance- 2 dominating set among all the vertices within the distance of ninety kilometers are sought. Figure 7 and Figure 8 give a minimal and minimum distance-2 dominating sets by using previous algorithms.

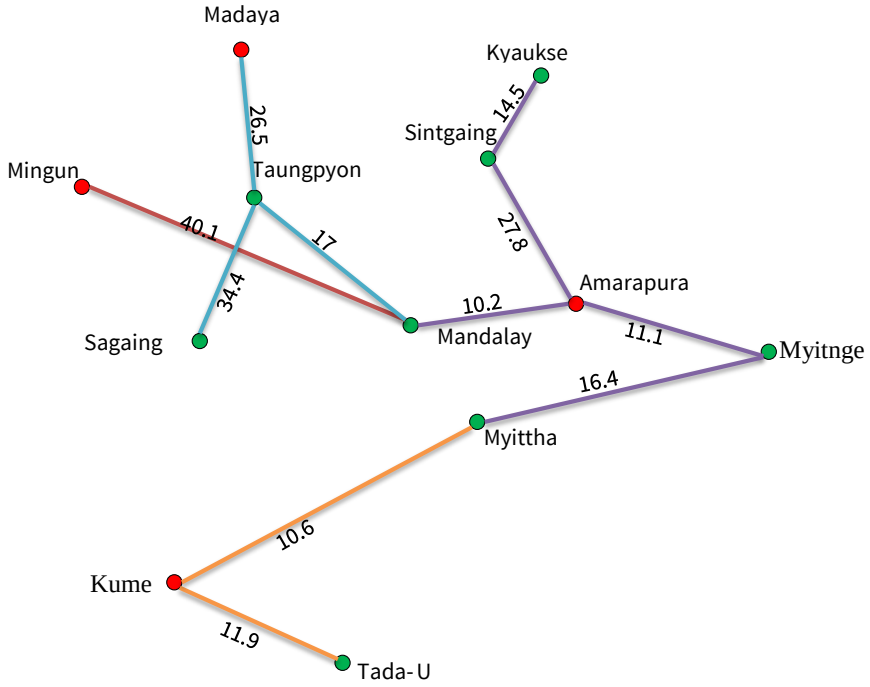

Figure 7. A minimal distance-2 dominating set {Madaya, Mingun, Amarapura,Kume} and domination number is 4.

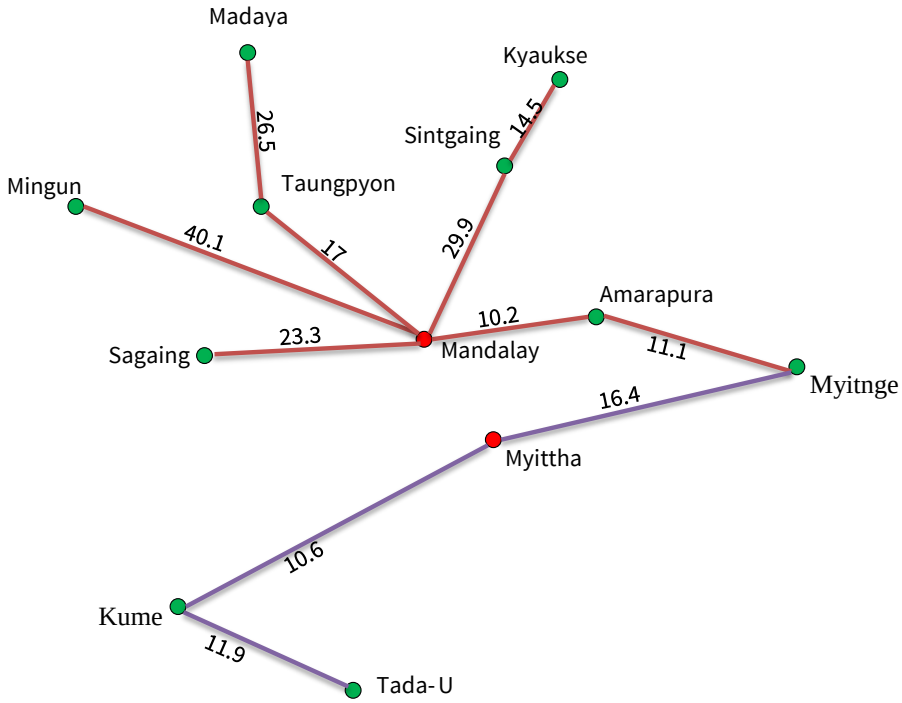

Figure 8. The minimum distance – 2 dominating set {Mandalay, Myittha} and domination number is 2.

4.2 Computer Networks

A computer network is a set of devices such as printers, routers, server, etc., that are connected. The computer or other computing devices are the vertices of the network and the connection cables are the edges. The simple computers network described in Figure 9 may be the computer network of a small business.

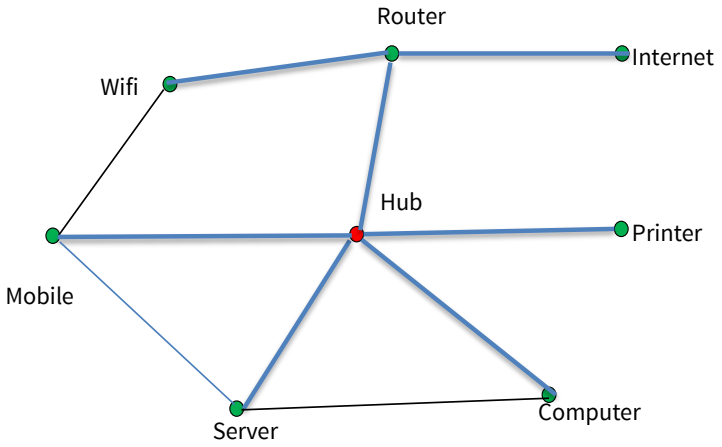

Figure 9. A simple computer network and minimum distance-2 dominating set is {Hub}.

4.3 Social Networks

Nowadays, Facebook is the most familiar social network. The concept of graph theory is used in Facebook with each person as vertices and every like, share, comment, tag as edges. Figure 10 shows the relationship of my Facebook network. The vertices represent names and they are joined by edges. From this Figure 10, we obtained the minimum dominating set.

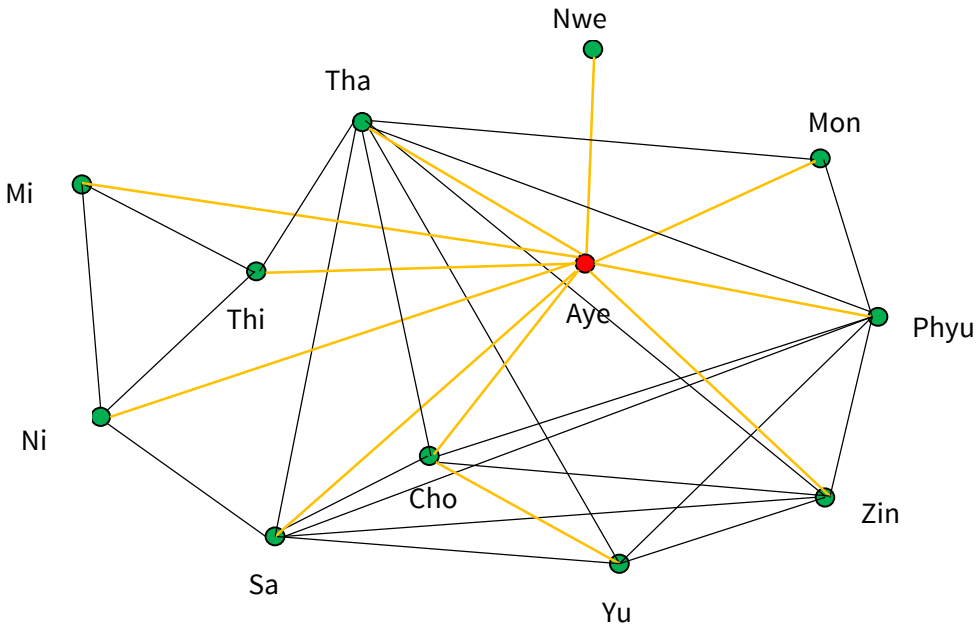

Figure 10. Relationship of Facebook Network and minimum distance-2 dominating set is {Aye}.

Conclusions

In this paper, the useful algorithms of minimal and minimum distance-2 dominating sets in radio station, computer networks and social network have been discussed. From the above discussion, many bounds and exact values for some standard graph are obtained. In the future, it is hoped to extend the algorithm to an inverse distance-2 dominating set of a graph.

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